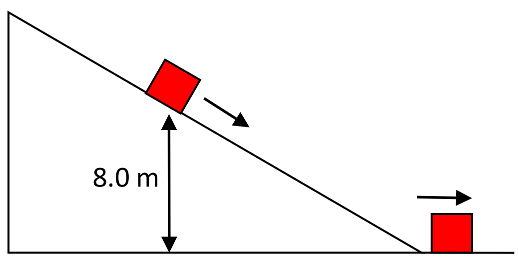
Work and energy

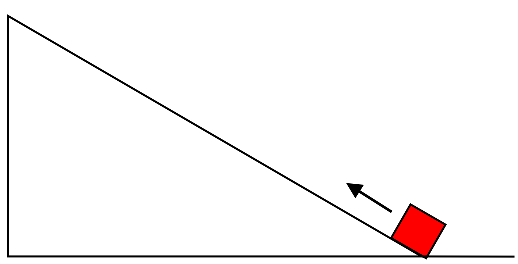
(a)
(i) State what the work done by the resultant resistive (i.e. non-conservative) force is equal to.
(ii) State what the work done by the resultant force is equal to.

(b) A block of mass 4.0 kg is given a tiny push, so it starts sliding down a rough inclined plane of angle 30° to the horizontal from a height of 8.0 m. When the block reaches level ground its speed is 5.1 m s^{-1}.



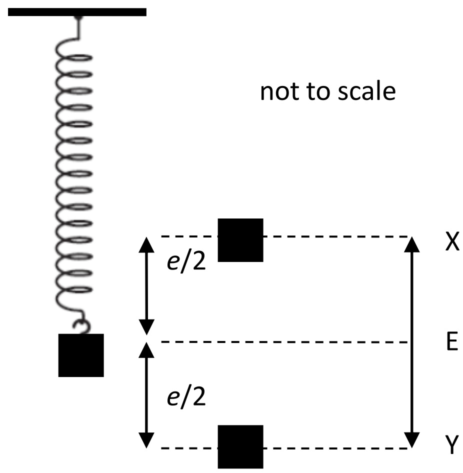
(i) Determine the frictional force acting on the block.
(ii) Estimate the coefficient of dynamic friction between the block and the plane.

(c) The block in (b) is now projected from the base of the same incline with speed 5.1 m s^{-1} up the incline. The static friction coefficient is 0.70.



(i) Discuss why the maximum height above level ground reached by the block will be less than 8.0 m.
(ii) Calculate this maximum height.
(iii) Discuss the subsequent motion of the block.

(d) A block of mass *m* is attached to the end of a vertical spring obeying Hooke's law. The mass is in equilibrium position E with the spring extended by a distance *e*. The spring is extended by an additional distance $\frac{e}{2}$ and is then released. The mass oscillates between the extreme positions X and Y.



(i) State Hooke's law.

(ii) Calculate the ratio of the elastic potential energy at Y to that at X.

(iii) Determine, in terms of m, g and e, the work done by the tension in the spring as the mass moves from X to Y.

(e) The engine of a car of mass 1200 kg develops a power of 22 kW when the car moves at constant speed 18 m s^{-1} on a horizontal road.

(i) Determine the resistive force acting on the car.

(ii) The car now begins to move up an inclined road that makes an angle of 5.0° to the horizontal. Determine the additional power that the engine must develop so that the car continues to move at the same constant speed of 18 m s^{-1}. The resistive force stays the same.

Answers

(a)

(i) The work done by the resultant resistive force on a system is equal to the change in the total mechanical energy of the system.

(ii) The work done by the resultant force on a system is equal to the change in the kinetic energy of the system.

(b)

(i) The change in the mechanical energy of the system is $\frac{1}{2}mv^2 - mgh = \frac{1}{2}\times 4.0\times 5.1^2 - 4.0\times 9.8\times 8.0 = -261.59\text{ J}$. The distance travelled along the inclined is d = 16 m. Hence, $fd\cos 180^\circ = -16f$. Thus, $-16f = -261.59 \Rightarrow f = 16.35 \approx 16\text{ N}$.

(ii) The normal force is $mg\cos 30^\circ = 4.0\times 9.8\times\cos 30^\circ = 33.95\text{ N}$, hence $\mu = \frac{16.35}{33.95} = 0.48$.

(c)

(i) On the way down the gravitational force was opposite to the frictional force. On the way up, both forces oppose the motion resulting in a greater deceleration and so a smaller height.

(ii) The change in the mechanical energy of the system is $mgh - \frac{1}{2}mv^2 = 4.0\times 9.8\times h - \frac{1}{2}\times 4.0\times 5.1^2 = 39.2h - 52.02$. The distance travelled along the inclined is 2h. The frictional force is $f = 16.35\text{ N}$. Hence, $-16.35\times 2h = 39.2h - 52.02$ leading to $h = 0.72\text{ m}$.

(iii) The maximum frictional force between the block and the inclined plane is $f_{max} = \mu mg\cos 30^\circ = 0.70\times 4.0\times 9.8\times\cos 30^\circ = 23.8\text{ N}$. The component of the weight down the plane is $mg\sin 30^\circ = 4.0\times 9.8\times\sin 30^\circ = 19.6\text{ N}$. Hence the block will stay at rest on the inclined plane with a static frictional force equal to 19.6 N balancing the component of the weight down the plane.

(d) The magnitude of the tension force in a spring is proportional to the extension.

(i) At Y the extension is $e + \frac{e}{2} = \frac{3e}{2}$. At X it is $e - \frac{e}{2} = \frac{e}{2}$. The ratio of elastic potential energies is then $\frac{\frac{1}{2}k(\frac{3e}{2})^2}{\frac{1}{2}k(\frac{e}{2})^2} = 9$.

(ii) The change in kinetic energy from X to Y is zero (= 0 – 0). This is equal to the work done by the resultant force i.e. $0 = W_T + W_{mg} \Rightarrow W_T = -W_{mg} = -mge$.

OR

The work done by the tension is the negative of the change in the elastic potential energy of the spring. The change in elastic energy is $\Delta E_e = \frac{1}{2}k(\frac{3e}{2})^2 - \frac{1}{2}k(\frac{e}{2})^2 = \frac{9ke^2}{8} - \frac{ke^2}{8} = ke^2$. But at equilibrium we have $ke = mg \Rightarrow k = \frac{mg}{e}$. Hence $\Delta E_e = ke^2 = \frac{mg}{e}e^2 = mge$. Hence the work done by the tension is $W_T = -mge$. We should expect a negative answer since from X to Y the tension is pointing upward whereas the displacement is downward.

(e)

(i) The force F of the engine pushing the car forward is given by $P = Fv \Rightarrow F = \frac{P}{v} = \frac{22\times 10^3}{18} = 1.22\times 10^3\ \text{N}$. Since the velocity is constant, the acceleration is zero and hence the resistive force is equal to the engine force i.e. $1.2\times 10^3\ \text{N}$.

(ii) The engine force must now increase to $F' = F_{\text{resistive}} + mg\sin\theta = F + mg\sin\theta$. The additional power will then be $(mg\sin\theta)v = (1200\times 9.8\times \sin 5.0^\circ)\times 18 = 1.845\times 10^4\ \text{W} \approx 18\ \text{kW}$.